INDOOR TRACK DESIGN

BANKING DESIGN PRINCIPLES

The centrifugal force on a body moving around a track curve of radius R is

$$F = mv^2/R$$

whereas on the straight section of track there is no centrifugal force acting on a body.

The centrifugal forces on a curve are counterbalanced by the athlete leaning into the curve, the effect of banking and the grip of the athlete's shoes on the track surface. The counterbalancing force of athlete lean angle from perpendicular of B degrees and the banking angle of A degrees is mg tan (A + B). It would not be appropriate to rely on a sideways thrust on the athlete's shoes. Taking all these factors into account

$$mg \tan(A + B) = mv^2/R$$
$$\tan(A + B) = v^2/gR$$

The angle of banking has to take into account that athletes will run at different velocities in different events. A top 200m runner could have a velocity of up to 10m/s whilst a female 5000m runner might only be running at 5m/s. Also athletes in the outer lanes have the advantage that the radius is larger and the centrifugal force on them is less.

The radius on which an athlete is running in lane 1 of the IAAF standard indoor track is 17.5m. Using a velocity of 10m/s, (A + B) = 30.22 degrees. For the same athlete running in lane 6 with a radius of say 21.831m, A + B = 25.03 degrees. For an athlete running at 5 m/s in lane 1, A + B = 8.29 degrees. Therefore a banking angle of about 10 degrees might be appropriate.

One might draw a parallel with a 10 m/s runner in a 200m race on a standard 400m track with the lane 1 running line being on a radius of 36.8m. A + B = 15.48 degrees. As A is only 0.57 degrees the lean of the athlete must be 14.91 degrees. If the banking of an indoor track were taken as 10 degrees the lean required for a 10m/s runner would be approximately 20 degrees to counteract centrifugal forces. Is that too much for comfort and is it more likely to cause injury?

Rickard Bryborn of the Swedish Athletic Federation has prepared the attached table showing the leaning of an athlete compared to the banked surface for a range of radii and banking angles.

The other banking consideration is that IAAF Rule 214.7 limits the height of banking at the most advanced staggered start line for 400/800m races in the outside lane to 80cm or one half of the maximum height of the banking at the peak of the bend (whichever is greater). With the present standard IAAF track the lane 6 200/400m start lines are on the bend with constant inclination of 10.0925 degrees at the limit of height allowed under Rule 214.7 unless the finish line is shifted along the main straight. Therefore again a limit of banking inclination of about 10 degrees would seem to give a straightforward layout result.

The athlete in the outer lane has the maximum rise and fall to negotiate. The angle of the banking and the width of the lanes determine the maximum rise. Swedish experience has shown that the percentage rate of rise should not exceed 5%. A table of rises provided by Rickard Bryborn is attached.

For a particular maximum height of banking the rate of rise will increase as the vertical radius of transition is increased. Here again experience has shown that this radius should not be less than 50m. However, even larger vertical radii may be appropriate.
Adjustable Banking

It is possible to construct a track with hydraulically adjustable banking such that for different speed races the banking angle can be altered to suit. The track markings must be measured at the maximum banking angle determined as appropriate for the 200m event. The measurements would then be longer for lower banking angles as the radius of the measuring line would increase. The increase in length would depend on the radius of the track, the width of the lanes and the angle change. To give an extreme example if a 17.5m radius track with 900mm wide lanes was lowered from 18 degrees to 8 degrees, the increase in length of the 200m race in lane 6 would be 880mm with proportionally lesser increase in other lanes. Lane 1 would be virtually unaffected. A practical consideration is whether the construction could accept an 880mm change in length.

It might be an advantage for 5000m runners to have about an 8 degree banking.

For certification of such a track there must be a clear requirement that the certified banking angle is used when the 200, 400 and 800m, and relays are conducted on the track

TRANSITIONS

Transition curves are inserted to smooth the passage from the straight with infinite radius to a curve of finite radius. A curve with a continually varying radius is a spiral. The centrifugal force on an athlete will gradually increase around the transition spiral as the radius decreases. A spiral on which the centrifugal force increases uniformly as the distance travelled along the spiral is an Euler spiral also known as a clothoid.

For the clothoid $Ø = (kl)^2/2RLs$

where Ø = spiral angle at any point on the spiral
k = constant
l = distance along the spiral
R = radius of the circular curve
Ls = length of spiral

The equations for the clothoid are somewhat complex but can be expressed in terms of the angle of the tangent to the spiral Ø at a point x,y on the spiral. The Fresnel integrals are used in physical optics and are well tabulated. Tables of standard offsets for a given curve radius and transition length are published.

An approximation of the clothoid is a cubic spiral derived by considering only the leading terms of the expansion of sin Ø and cos Ø such that $y = l^3/6RLs$ and $x = l(1-l^4/40R^2Ls^2)$

Some of the properties of the clothoid are:

The spiral angle at any point is $(l^2/Ls^{2)}$ x A where,

> l = distance along spiral
> Ls = total length of spiral
> A = maximum spiral angle

The angle of the tangent to the transition spiral at the midpoint of the transition is one quarter of the maximum spiral angle.

The circular curve is moved in a small amount so that the transition spiral joins it smoothly. This shift (S) may be calculated from:

> $S = Ls^2/24R$, where R = radius of the circular curve

To counteract the increasing centrifugal force on the athlete along the transition the increase in banking should also be uniform until the maximum banking angle is achieved. This does not seem to be the situation proposed in the Manual Figure 8.2.1.1a with the transition curve "3" followed by a ascending track section "4" with the same radius as the bend with constant inclination. I propose that the transition be as long as the vertical transition. . Note IAAF Rule 213.4 does not allow the vertical transition to start more than 5m into the straight. The reasoning for this is not clear as when the radius is smaller than the IAAF optimum radius the straight is longer and there is a need for a longer transition. The table below gives some approximate straight lengths for various radii

Running Line Radius m	**Length of 2 Full Bends m**	**Length of Straight w/o transition m**	**One Straight less 2 No. 5m Transition on Straight m**
19.5	122.522	38.739	28.738
17.5	109.956	45.022	35.022
15.5	97.389	51.305	41.305
13.5	84.823	57.588	47.588
11.5	72.257	63.872	53.872

Slightly less than half of the spiral transition curve occurs on the straight before the circular curve tangent point. Thus a pure clothoid spiral in those circumstances can only be slightly longer than 10 metres.

Whilst the kerb or lane 1 marking can be set out with a true transition spiral, all the other lane markings will not be true spirals. Each lane has to be set out from the kerb on a radius from the kerb with the coordinates x, y, and z determined so that the lane width is maintained on the banked track. Mattias Bryborn of the Swedish Federation has provided mathematical calculations for this situation. Using a computer three-dimensional coordinates can be readily calculated.

Another way of handling the transition is to make it three or four sections of radii gradually decreasing from a very large radius to a radius closer to the bend radius over the full vertical transition range. This is a very practical solution to the transition problem that has been adopted by several manufacturers.

The length of each lane in the transition can be accurately calculated using appropriate mathematical formulae for small discrete intervals using a computer program. However, the length of each lane transition would be quicker measured by steel tape when the track is being check measured.

Determination of the Appropriate Spiral Length

The change of centrifugal force on the athlete running on the clothoid transition is uniform when the athlete is running with uniform velocity v. Let a = uniform rate of change of radial acceleration in m/s^3 on the transition.

$$a = \text{radial acceleration}/\ t = v^2/rt$$

Where t = time for athlete to run a transition of length Ls = Ls/v

Therefore $a = v^3/rLs$; and $Ls = v^3/ra$

In road design a is limited to a maximum of about $2.1m/s^3$ on very tight turns at relatively low vehicle speeds.

Using the IAAF standard track for an athlete running at 10m/s in lane 1 radius r = 17.5m $a = 1000/17.5 \times 10.108 = 5.65m/s^3$.

If a maximum value of "a" is determined it would be possible to dictate the minimum length of transition for each radius of the banked track by a simple formula.

In simple terms the smaller the curve radius the longer the transition required and this can be catered for because the straights are somewhat longer. However, Rule 213.4 limits the use of the straight for the transition and as stated above the length of the transition is therefore limited to just over 10 metre length no matter what is the bend radius.

Recommendations

A Working Group of indoor track design and construction specialists be appointed to rewrite Chapter 8 of the IAAF Track & Field Facilities Manual. taking account of the issues noted above.

The Working Group to recommend any necessary changes to IAAF Handbook Section VI - Indoor Competitions so as to allow more appropriate designs.

Denis Wilson
10 February 2003

BANKING DESIGN PRINCIPLES

Explanations

R	Radius	Horizontal force	F1	m * v2/R
v	Velocity	Vertical force	F2	m * g
m	Weight			
a	Angle compared to vertical line	angle (tg "a")	F1/ F2	
b	Angle compared to horizontal surface	angle "b"	(90-a) degree	
c	Angle compared to banked surface			

TABLE showing the leaning of a body compared to the floor

Velocity	Radius of running line	F1	F2	F1/F2	Angle "a"	Angle "b"	Angle of banking	Angle "c"
m/s	m				compared to vertical line	compared to horizontal surface	Chosen	compared to banked surface
10	11.5	8.70	9.81	0.89	41.6	48.4	18	66.4
10	12.5	8.00	9.81	0.82	39.2	50.8	17	67.8
10	13.5	7.41	9.81	0.76	37.1	52.9	17	69.9
10	14.5	6.90	9.81	0.70	35.1	54.9	15.5	70.4
10	15.5	6.45	9.81	0.66	33.3	56.7	13.5	70.2
10	16.5	6.06	9.81	0.62	31.7	58.3	12.5	70.8
10	17.5	5.71	9.81	0.58	30.2	59.8	10	69.8
10	18.5	5.41	9.81	0.55	28.9	61.1	10	71.1
10	19.5	5.13	9.81	0.52	27.6	62.4	10	72.4
10	20.5	4.88	9.81	0.50	26.4	63.6	10	73.6
10	36.8	2.72	9.81	0.28	15.5	74.5	0.5	75.0
5	11.5	2.17	9.81	0.22	12.5	77.5	18	95.5
5	12.5	2.00	9.81	0.20	11.5	78.5	17	95.5
5	13.5	1.85	9.81	0.19	10.7	79.3	16	95.3

5	14.5	1.72	9.81	0.18	10.0	80.0	15	95.0
5	15.5	1.61	9.81	0.16	9.3	80.7	13	93.7
5	16.5	1.52	9.81	0.15	8.8	81.2	12.5	93.7
5	17.5	1.43	9.81	0.15	8.3	81.7	10	91.7
5	18.5	1.35	9.81	0.14	7.8	82.2	10	92.2
5	19.5	1.28	9.81	0.13	7.4	82.6	10	92.6
5	20.5	1.22	9.81	0.12	7.1	82.9	10	92.9
5	36.8	0.68	9.81	0.07	4.0	86.0	0.5	86.5

With the chosen angles the leaning of the body is almost the same in the most common range of the radius (14,5 - 19,5m)
The length of radius has been chosen from experiences from existing Swedish arenas

PROPOSAL for measurements

Radius	11,5m	13,5 m	14,5 m	15,5 m	16,5 m	17,5 m	18,5 m	20,5 m
Banking	18 degrees	16	15	13	11.5	10	10	10

2002-05-21
Swedish Athletic Association
Rickard Bryborn

INDOOR TRACKS BANKING TRANSITION LENGTH

The transition length on the running line of the outer lane is calculated at 5.0% and 4.5% of the track rise at the end of the transition

No. Lanes	Lane Width	Banking Angle	Track Rise	Transition Length 5%	Transition Length 4.5%
6	0.9	18	1.452	29.048	32.28
6	0.9	17	1.374	27.483	30.54
6	0.9	16	1.295	25.910	28.79
6	0.9	15	1.216	24.329	27.03
6	0.9	13	1.057	21.145	23.49
6	0.9	11.5	0.937	18.741	20.82
6	0.9	10	0.816	16.323	18.14
6	1.0	18	1.607	32.138	35.71
6	1.0	17	1.520	30.407	33.79
6	1.0	16	1.433	28.666	31.85
6	1.0	15	1.346	26.917	29.91
6	1.0	13	1.170	23.395	25.99
6	1.0	11.5	1.037	20.734	23.04
6	1.0	10	0.903	18.059	20.07
6	1.1	18	1.761	35.228	39.14
6	1.1	17	1.667	33.330	37.03
6	1.1	16	1.571	31.423	34.91
6	1.1	15	1.475	29.505	32.78
6	1.1	13	1.282	25.644	28.49
6	1.1	11.5	1.136	22.728	25.25
6	1.1	10	0.990	19.796	22.00
4	1.1	18	1.082	21.631	24.03
4	1.1	17	1.023	20.466	22.74
4	1.1	16	0.965	19.295	21.44
4	1.1	15	0.906	18.117	20.13
4	1.1	13	0.787	15.747	17.50
4	1.1	11.5	0.698	13.956	15.51
4	1.1	10	0.608	12.155	13.51